Chapter 10

Run-off triangles

0 Introduction

0.1 Run-off triangles

- Run-off triangles are used in general insurance to forecast future claim numbers and amounts.

- In this chapter we will look at four standard methods for projecting run-off triangles
  - the basic chain ladder method
  - inflation-adjusted chain ladder method
  - the average cost per claim method
  - Bornhuetter-Ferguson method
• Run-off triangles usually arise particularly in non-life insurance where it may take some time after a loss until the full extent of the claims which have to be paid are known.

• It is important that the claims are attributed to the year in which the accident occurred.

• The insurance company needs to know how much it is liable to pay in claims so that it can calculate how much surplus it has made.
• It is clear that although the insurance company does not know the exact figure for total claims each year because there are delays in the claim totals, it must try to estimate that figure with as much confidence and accuracy as possible.

• The question that we shall attempt to answer in this chapter is:

    How much needs to be set aside now (as a reserve) to meet future payments to be made on claims that have arisen during some recent past period?
0.2 Types of reserves

- An *IBNR* claims reserve is required in respect of claims that have been incurred but not reported.
- An *outstanding reported* claims reserve is required in respect of claims that have been reported, but have not yet been closed.
0.3 Presentation of claims data

- Claims data will be presented as a triangle, which is the most commonly used method.
- The year in which the incident happened and the insurer was on risk is called the accident year.
- The number of years until a payment is made is called the development year.
- Claims data may be presented cumulatively, or on an individual year basis.
- See the Example on page 4.
Various alternative tabulations could have been used. For example:

- The cohorts could be defined by reporting year ("all claims reported in year X") or by written year ("all claims from policies written in year X").
- The origin years might be the company’s financial years or might be origin quarters or origin months.
- The entries in the table might show numbers of claims or estimated ultimate cost or claims-related expenses.
0.4 Estimating future claims

- The task is to decide the amounts yet to be paid in respect of the given accident years.
- The process to fill in the lower triangle by comparing present figures with past experience is the main object of this chapter.
1 Projections using development factors

1.1 Run-off patterns

- The basic assumption made in estimating outstanding claims concerns the run-off pattern.
- The simplest assumption is that payments will emerge in a similar way in each accident year.
- The proportionate increases in the known cumulative payments from one development year to the next can then be used to calculate the expected cumulative payments for future development years.
- The ratios that are used to project future claims are known as development factors.
- See the Example on pages 8-9.
1.2 A statistical model for run-off triangles

- It is helpful to have the following statistical model in the back of your mind when you are considering the various methods that may be used to complete the run-off triangle.

- See the general form of a run-off triangle on page 10.
1.3 The chain ladder method

- In this section we will carry out the calculations for completing the run-off triangle using the basic chain ladder method.
- The name given to this method presumably arises from the ladder-like operations which are chained over the development years.
- See the Example on pages 11-14.
1.4 Model checking

- The chain ladder technique is used primarily to estimate the development of cumulative claim payments.
- However, it is useful to check whether it fits reasonably with the claims data which have already been received.
- See the Example on pages 15-16.
1.5 Other methods of deriving development factors

- It is possible to adjust the calculated development factors in the light of other information.
- The development factors, either calculated directly from the data, or set using expert knowledge, are always used in the same way to estimate outstanding claim payments.
• **Example on page 17**
  Suppose that a new computer system is installed that speeds up the claims process substantially. If a run-off triangle method is applied to a period that spans the time both before and after the installation of the new system, the underlying development factor are likely to change when the system is installed. Because calendar years span the diagonals of the triangle, rather than horizontal lines, the effect of a change of this kind is likely to be unpredictable.

• When using run-off triangles, you should always be on the look out for factors that may distort the pattern of the run-off.

• The chain ladder method can also be applied to a triangle of loss ratio data rather than cumulative payments.
1.6 Assumptions underlying the method

- The chain ladder technique is based on the assumption that payments from each accident year will develop in the same way.

- Changes in the rate at which claims emerge can only be incorporated by "hand adjustment" of the development factors.

- The final assumption made when the chain ladder technique is used concerns inflation.

- It is assumed that weighted average past inflation will be repeated in the future.

- This is because claims inflation is one of the influences swept up within the projection factors.
Using the general statistical model, it can be seen that the basic chain ladder takes the form:

\[ C_{ij} = r_j s_i + e_{ij} \]

This might be an unrealistic assumption.

When considering inflation, it is important to bear in mind that it is claims inflation which is important. Thus, although a standard measure of overall inflation may be used, the inflation rate inherent in claims may be quite different.

– Inflation in the price of consumer goods
– Inflation in the cost of settling claims
– Special indices of inflation for things like the cost of repairing vehicles.
2 Adjusting for inflation

2.1 The inflation adjusted chain ladder method

2.1.1 Dealing with past inflation

• Claims inflation will affect the payments in the run-off triangle by calendar year of payment.

• The inflation adjusted chain ladder method works by adjusting the figures in the triangle to allow for the effects of inflation.

• In the model considered here, it will be assumed that claims inflation is at the same annual rate for all claims within a particular calendar year of payment.

• When adjusting far inflation, it is the payments in each calendar year which need to be considered, rather than cumulative totals.
2.1.2 Dealing with future inflation

• The predictions of cumulative payments do not, however, take account of future inflation.

• In order to forecast the actual payments, an assumed rate of future inflation will be needed.

• Again, it is necessary to convert to non-cumulative data rather than the cumulative totals before adjusting these for future inflation in a similar way to that used when dealing with past inflation.
2.1.3 Assumptions underlying the method

- The key assumption underlying this method is that, for each origin year, the amount of claims paid, in real terms, in each development year is a constant proportion of the total claims, in real terms, from that origin year.

- Explicit assumptions are made for both past and future claims inflation.

- Therefore, using the general statistical model, the inflation adjusted chain ladder method takes the form:

\[ C_{ij} = r_j s_i x_{ij} + e_{ij} \]
3 The average cost per claim method

• This method, considers separately the two key elements of total claim amounts, i.e. the number of claims and the average amounts of the claims.

3.1 Description of method

• First stage
  – This method requires development tables for both total claim amounts and claim numbers.
  – We normally use cumulative figures.
  – A third development table, of the average claim amounts, is then formed by dividing the figures in the corresponding cells of the first two tables.
3 THE AVERAGE COST PER CLAIM METHOD

- Second stage
  - Project figures in the average claims and number of claims tables, using either grossing-up factors or development factors.
  - A grossing-up factor gives the proportion of the ultimate claim amount that has been paid so far.

- Last stage
  - The projected ultimate claims can be calculated by multiplying together for each accident year the projected figures for the average claim amounts and claim numbers.
  - A reserve can then be calculated by subtracting all payments to date in respect of claims relating to the data in the table.
3.2 Application of the method (To be omitted)

3.3 Example

- Example on pages 26-29.

3.4 Assumptions underlying the method

- As there is no unique way of defining the method, there is no unique set of assumptions.
- In general terms, however, there are the assumptions that for each origin year, both the number and average amount of claims relating to each development year are constant proportions of the totals from that origin year.
4 Loss ratios

- The ratio of incurred claims to earned premiums over a defined period is called the loss ratio.

\[
\text{loss ratio} = \frac{\text{incurred claims}}{\text{earned premiums}}
\]

- A loss ratio based on trends of past data, underwriters' views, or market data, could be used as a basis for an estimate of the eventual loss and hence the outstanding claims.

- This concept of a loss ratio will play an important part in the Bornhuetter-Ferguson method.
5 The Bornhuetter-Ferguson method

5.1 Concept of the Bornhuetter-Ferguson method

- The Bornhuetter-Ferguson method combines the estimated loss ratio with a projection method.

- The concepts behind the method are:
  - That whatever claims have already developed in relation to a given origin year, the future development pattern will follow that experienced for other origin years.
  - The past development for a given origin year does not necessarily provide a better clue to future claims than the more general loss ratio.
5.2 Description of the method

• The future claims development can be expressed as:

\[
\text{Future claims development} = \text{Premium} \times \text{Estimated Loss Ratio} \times (1 - 1/f)
\]

• As the final estimate of the ultimate loss is based on observed data and an initial estimate ignoring the observations, this method could be viewed as using a Bayesian approach.
• Using B-F method, each year’s revised ultimate loss can be expressed as a combination of the chain ladder ultimate loss (CL) and the independent loss ratio ultimate loss (LR).

• Using a credibility factor (Z), this can be expressed as:

\[ \text{B-F revised ultimate loss} = Z \times CL + (1 - Z) \times LR \]

where

\[ Z = A/CL \]

and \( A \) is the actual claims developed to date figure (\textit{i.e.} the figure from the latest lead diagonal in the run-off triangle.)
• So, $Z = A/CL$ is just the inverse of the ultimate development factor ($f$) for the year.

• And

$$\text{B-F revised ultimate loss} = A + (1 - 1/f) \times LR$$
5.3 Application of the method (To be omitted)

5.4 Example

- Example on pages 35-38.

5.5 Assumptions underlying the method (To be omitted)

5.6 Grossing-up factors versus development factors (To be omitted)