Advanced Macroeconomics I
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Assignment 2

Note: You may use either English or Chinese to answer the questions.

Question 1 (5 points)

Suppose there is a large number of identical agents with preferences represented by

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln c_t$$

where $c_t$ is consumption in period $t$. Agents can trade one-period real bonds. The gross return on a bond purchased in period $t$, denoted $R_t$, is known at the time of purchase (i.e. the bonds are risk-free). Therefore, the price of a bond is $R_t^{-1}$.

The constraints faced by a representative agent are

$$c_t + L_t R_t^{-1} = A_t, \quad A_{t+1} = L_t, \quad A_0 \text{ given}$$

where $A_t$ is wealth at the beginning of period $t$ and $L_t$ is the holding of one-period bonds. Assume that $R_t$ is identically and independently distributed and is such that $E R_t < 1/\beta$.

(a) (1 pts) Using $L_t$ as the control variable, write down Bellman equation corresponding to the representative agent’s optimization problem.

(b) (2 pts) Use dynamic programming to derive the Euler equation corresponding to the representative agent’s problem. Interpret this Euler equation.

(c) (2 pts) Use the guess $L_t = \gamma R_t A_t$ to solve for the policy function for bond holdings $L_t$ (and consumption).
Question 2 (5 points)

Consider a simple “real business cycles” model where a planner maximizes utility

\[
E_0 \sum_{t=1}^{\infty} \beta^t [\ln c_t + \gamma \ln (1 - n_t)], \quad 0 < \beta < 1, \quad \gamma > 0
\]

where \(c_t\) denotes consumption and \(n_t\) hours worked. The maximization is subject to the resource constraint

\[
c_t + x_t = z_t k_t^{\alpha} n_t^{1-\alpha}, \quad 0 < \alpha < 1
\]

and the transition equation for capital

\[
k_{t+1} = (1 - \delta) k_t + x_t \quad 0 \leq \delta < 1.
\]

\(k_t\) denotes the capital stock installed at the beginning of period \(t\) and \(z_t\) is a stochastic disturbance interpreted as a productivity shock. \(\delta\) is the capital depreciation rate. Assume that

\[\ln z_t \sim iid(0, \sigma_z^2).\]

(a) (1 pts) What are the state variables and the control variables for this problem? What is Bellman equation corresponding to this problem?

(b) (2 pts) Using dynamic programming, derive the Euler equation for capital and the first-order condition for hours worked (substitute out any Lagrange multiplier). Interpret these equations.

(c) (1.5 pts) Now assume that \(\delta = 1\). Using the guess and verify method, show that the policy function for hours worked is time-invariant and that the policy function for investment is linear in output. (Hint: Guess \(k_{t+1} = \eta z_t k_t^\alpha n_t^{1-\alpha} = \eta y_t.\))

(d) (0.5 pts) In light of the results found in part (b), discuss the ability of this model to generate realistic fluctuations in hours worked.