Advanced Macroeconomics I

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Assignment 1

Note: You may use either English or Chinese to answer the questions.

Question 1 (3 points, 1 point for each part)

This question seeks to clarify the concept of expectations, conditional and unconditional.

(a) Consider a two-period model where consumption in period $t$ is a random variable denoted $C_t$, for $t = 0, 1$. For simplicity, assume that $C_t$ can take on two values, $c_t$ and $\bar{c}_t$, for $t = 0, 1$. Show that when $C_t$ is identically and independently distributed over time

$$E_0(C_1) = E[C_1].$$

(b) Consider a two-period model where consumption in period $t$ is a random variable denoted $C_t$, for $t = 0, 1$. For simplicity, assume that $C_t$ can take on two values, $c_t$ and $\bar{c}_t$, for $t = 0, 1$. Show that

$$E[E_0(C_1)] = E[C_1].$$

(c) Consider a three-period model where a consumer seeks to maximize her lifetime expected utility

$$E_0 \sum_{t=0}^{2} \beta^t u(C_t) = u(c_0) + \beta [u(\xi_1) \cdot Pr(C_1 = \xi_1 | c_0) + u(\tau_1) \cdot Pr(C_1 = \tau_1 | c_0)]$$

$$+ \beta^2 [u(\xi_2) \cdot Pr(C_2 = \xi_2 | c_0) + u(\tau_2) \cdot Pr(C_2 = \tau_2 | c_0)]$$

where $C_1$ and $C_2$ are random variables that can take on only two values, $\xi_t$ and $\tau_t$, for $t = 1, 2$. How would you calculate the conditional probabilities $Pr(C_2 = \xi_2 | c_0)$ and $Pr(C_2 = \tau_2 | c_0)$?
Question 2 (2 points)

Consider the following model of aggregate consumption:

\[ c_t = E_t (y_{t+1}^\beta), \quad \beta \in (0, 1) \]

where \( c_t \) is the level of consumption this period, and \( y_{t+1} \) is the level of income next period. \( y_t \) is a random variable.

Professor N. Aive proposes to test this model by following these steps:

(i) collect data on \( y \) and \( c \), calculate the average of each series, denoted \( \bar{c} \) and \( \bar{y} \).

(ii) find an estimate of \( \beta \), denoted \( \hat{\beta} \), as the value that solves the following nonlinear equation:

\[ \bar{c} = \bar{y}^{\hat{\beta}} \]

(iii) test the theory by comparing the sequence \( \{c_t\} \) with the sequence \( \{y_t^{\hat{\beta}}\} \).

Professor J. Ensen argues that the latter sequence will systematically underpredict the value of the former sequence. Which scholar is correct and why?

(Hint: Use Jensen’s Inequality to solve this problem. If you are not familiar with this useful tool, check the notes of the first lecture.)
**Question 3** (5 points, 1 points for each part)

Consider an extension of the two-period endowment economy studied in class where agents live for four periods. A representative agent has lifetime expected utility

\[
E_1 U = \ln(c_1) + 0.96E_1 \ln(c_2) + 0.96^2E_1 \ln(c_3) + 0.96^3E_1 \ln(c_4)
\]

where \(c_t\) denotes consumption in period \(t\). Each agent receives an endowment \(y_t\) in period \(t\).

(a) Let \(q^S_{bt}\) denote the price of a one-period bond purchased in period \(t\) and paying one unit of consumption in period \(t + 1\). State (no deviations nor explanations required) an expression relating the price \(q^S_{b1}\) to \(y_1\) and \(y_2\).

(b) Let \(q^M_{bt}\) denote the price of a two-period bond purchased in period \(t\) and paying one unit of consumption in period \(t + 2\). State (no deviations nor explanations required) an expression relating the price \(q^M_{b1}\) to \(y_1\) and \(y_3\).

(C) Let \(q^L_{bt}\) denote the price of a three-period bond purchased in period \(t\) and paying one unit of consumption in period \(t + 3\). State (no deviations nor explanations required) an expression relating the price \(q^L_{b1}\) to \(y_1\) and \(y_4\).

(d) Consider the following contract. In period 1, buyer and seller agree on the following: (1) in period 3, the buyer has to pay \(f\) units of consumption to the seller; (2) in period 4, the seller has to pay 1 unit of consumption to the buyer. Explain what are the utility costs and benefits to the buyer of signing the above described contract. What is the equation relating the price \(f\) to endowments?

(e) Using your answer from part (d), derive an expression relating \(f\), \(q^L_{b1}\) and \(q^M_{b1}\) and briefly explain why this equation has to hold in equilibrium.