Introduction to Game Theory & Information Economics

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Main textbook Martin Osborne (2004)

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Lecture 1: Introduction

1.1 OVERVIEW

• example: auction

• purpose: understanding regularity

• method: like geometry and physics

• history

• math level: high school

1.2 DECISION THEORY UNDER CERTAINTY

it only has one decision-maker
some basics

- pick your favorite from a basket of fruits

- outcome set $A$

- preference relation: (strictly) prefer, indifferent

Def: rational preference is complete and transitive.

Def: rational decision-maker chooses $a \in A$ s.t. $\forall b \in A$, the agent prefers $a$ over $b$.

examples include consumer theory, etc.

Q: What is $A$ in consumer theory?
utility/payoff function is more convenient.

Def: A utility function $u()$ is consistent with a preference relation if, $\forall a, b \in A$, $a$ is preferred over $b$ iff $u(a) > u(b)$.

example: $A = \{a(ptide), b(anana), o(range)\}$, $b$ preferred over $a$, while $a$ preferred over $o$, utility function $f(a) = 1, f(b) = 2, f(o) = 0$ is consistent with the above preference relation.

how about $g(a) = 1.5, g(b) = 2, g(o) = 0$?

Theorem: For any preference relation over a finite set $A$, there exists a consistent utility function $u()$.

proof?

Does $2u()$ represents the above preference relation?

Lecture 2: Solution to a Game
2.1 WHAT IS A GAME?

Play a simple class experiment. Ask several pairs of students to write a name (one of the partners in the pair) in separation and reward them if their names coincide.

Some comments about the difference between decision and game:

- decision-maker v.s. players

- preference relation (payoff function) still defined over outcome set A

- in decision problem outcome = action. In a game outcome corresponds to all the actions (action profile)
timing: simultaneous-move (strategic) game v.s. dynamic game.

information: perfect v.s. imperfect

What is player’s belief?

common-knowledge assumption

2.2 NASH EQUILIBRIUM (NE)

The players are still rationally “choosing” the best outcome

The available outcomes are determined by other player(s)

But players do not know each other’s actions

Players need to form beliefs about each other’s actions
John Nash’s additional restriction: beliefs should be correct

what does it mean here?

Sun Tse on war strategy is based upon cheating: story of Tian Ji racing

A more familiar definition of NE (below) is equivalent.

Def: A Nash Equilibrium is an action profile $a^*$ with the property that no player $i$ can do better by choosing an action different from $a^*_i$, given that every other player $j$ adheres to $a^*_j$ (notation: given $a^*_{-i}$).

interpretation: social norm is stable, no one wants unilateral deviation.

For example, social institutions are NE
NE does not tell you how correct beliefs are formed

It tells you that incorrect beliefs are not stable in the long term

Correct beliefs (expectations) might be formed through collective experiences.

Or through deductive logic (eg, prisoner’s dilemma)

Prisoner’s Dilemma

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<th>N</th>
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<td>2, 2</td>
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<tr>
<td>Y</td>
<td>3, 0</td>
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2.3 SOME EXAMPLES

To get NE, we can check belief consistency for the following outcomes.
Matching Pennies (conflict)

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Meeting (coordination)

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<td>B</td>
<td>0, 0</td>
<td>1, 1</td>
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Games of pure conflict are first studied by mathematicians.

It has no pure NE.

But many real-world games are mixture of conflict and coordination.

Lecture 3: Coordination & Equilibrium Selection
Coordination games include the following

Language: the form is not important, the meaning is.

Driving on the left or right is not important, but we need to pick one.

Matching buyer with seller is important, for that purpose anything will do.

3.1 FOCAL POINT

invite everyone (or a large number) to participate in the previous name a name game.

example: star effect

a simple experiment of naming a star’s name.

We run 2 class experiments of the following
Rules of experiments: listen carefully, keep quiet, no communication.

The SUPER GIRL contest, introduce background and top-4 stars.

1. First students are asked to write down the name of one star.

Hand in and count.

2. Tell students they are randomly matched into pairs

Again students are asked to write down one name of the star.

If both students in the same pair name the same star, they win.

Hand in and count.
The votes should be more equally distributed in the first experiment.

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<th>4</th>
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<td>1,1</td>
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<td>4</td>
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<td>0,0</td>
<td>1,1</td>
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The first experiment is not a game.

In the second experiment more people (should) pick (1,1) equilibrium.

As experiments might show, popularity is not necessarily a good prediction of the coordination game.

Here (1,1) is a FOCAL POINT of coordination (best star).
Note: (4,4) can also be a focal point (the only non-top-3 star).

Super-Girl cafe: suppose each star opens a cafe of her name in Shanghai, which one will be more crowded?

Imagine two lonely young people are making choices after dinner at different corners of the city.

Comments:

Many economic and social activities are (more) valuable only if carried out by a group of people together

People pick one particular outcome because it is a focal point for coordination, not necessarily because they have an inherent interest in it,
nor because the object has some inherent good quality.

Important practical question: who are the players?

Among fans of the No.2 star (in our class), the focal point is the No.2 star, not the No.1 star.

The same people under different circumstances might treat different objects as focal points.

With different people you go to different places for the night. You may want to watch different TV with different people.

To market a super girl (or an instant messenger service, a TV channel, etc), you need to make her an object of focus among a particular group of people (maybe even at particular time and location).
Market share or advertisement is not necessarily an indicator of product strength. Easy comes, easy goes.

Implication for bar business: bar should be small and have a theme.

Examples also include the following:

Entertainment (how people spend the night together? which TV channel to watch together?)

Communication technology (QQ or MSN or Yahoo! Messenger)

Language: the form is not important, the mutual meaning is.

Driving on the left or right is not important, but we need to pick one.
Matching buyer with seller is important, for that purpose anything will do. (where to buy a (fake) LV bag for 100RMB? xiangyang lu or qibao lu or other place, the location does not matter)

technical comment:

There are often many Nash equilibria.

The definition of NE says nothing about comparison and selection

We take two directions:

1. analytical (refining the model, how smart people guess each other)

2. empirical (using information outside the model, how ordinary people use cues)
Rest of the course includes the first approach.

This lecture mainly takes the second approach.

Coordination game is good for illustration.


3.2 RISK/PAYOFF DOMINANCE

stag hunt game with naming a name rewarding only your name game.

The following game is easy to play.

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<th>A</th>
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<tbody>
<tr>
<td>A</td>
<td>3,3</td>
<td>0,0</td>
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<tr>
<td>B</td>
<td>0,0</td>
<td>1,1</td>
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Most people will choose A.

But how about this one?

**Stag Hunt**

<table>
<thead>
<tr>
<th></th>
<th>stag</th>
<th>rabbit</th>
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<tbody>
<tr>
<td>stag</td>
<td>3, 3</td>
<td>0, 2</td>
</tr>
<tr>
<td>rabbit</td>
<td>2, 0</td>
<td>1, 1</td>
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Unlike driving on the left or right choice, in which left or right is not important, there are coordination situations in which the choice matters a lot.

Being on time for a group of people is a well known difficulty. Why?

In full-employment equilibria, workers are employed, they all spend, which creates demand for products and keeps them employed (good
macroeconomic outcome) → But if they (fear a recession and) cut back spending together, they will cause layoffs and real economic recessions (bad macroeconomic outcome)

Similar firms want to locate close to each other: industrial clustering in Zhejiang Province, electronic industry in Suzhou and Pearl River delta, film industry in Hollywood, IT industry in Silicon Valley, etc.

It is not an easy choice. Imagine two electronic firms considering investing in Suzhou in 1995. What if the other firms fail to invest when you do?

The main tradeoff is between payoff and risk. (Stag, Stag) is a better outcome for both players. But choosing stag is risky if the other player chooses rabbit.
3 measures payoff, while 0 measures risk.

If we increase 3 to, say, 10, holding everything else unchanged, we will observe more play of “stag.”

If we decrease 0 to, say, −5, holding everything else unchanged, we will observe more play of “rabbit.”

Harsanyi and Selten (1988) *A General Theory of Equilibrium Selection in Games*

3.3 PURE TIMING

We run 3 class experiments.

Half students are designated as row players, the other half as column players.
1. Each student is asked to submit a piece of paper with his/her hunting choice.

   Hand in and count.

2. The row players openly make the hunting choices first.

   Then the column players make the hunting choices.

3. The row players make the (secret) hunting choices first and hand in.

   Then the column players make the hunting choices.

   the effect of pure timing

   implication to game theory

3.4 FORWARD INDUCTION

Pure coordination game has symmetric payoff for all the equilibria.

No one cares which particular equilibrium is chosen, so long as people’s action is coordinated.

But for richer models, other incentives are added to coordination.

Naming a name game for two, but the named name gets higher pay.

Battle of sexes
This is still a coordination game.

Players have asymmetric payoffs in different equilibria.

In other words, row player prefers the equilibrium (A, A), while the column player prefers (B, B).

We run 3 class experiments.

Half students are designated as row players, the other half as column players.

1. Each students are asked to submit a piece of paper with his/her choice on BoS.
2. Row player has 3 choices – stoping the game, A, B. Column player’s choice is the same as before. They submit their choices at the same time. If the game is stopped, each player gets 1.5 and column player’s choice becomes irrelevant.

3. Row player has 3 choices – stoping the game, A, B. Column player’s choice is the same as before. They submit their choices at the same time. If the game is stopped, each player gets 0.5 and column player’s choice becomes irrelevant.

Battle of sexes (big outside option)

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<td>2,1</td>
<td>0,0</td>
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<tr>
<td>B</td>
<td>0,0</td>
<td>1,2</td>
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<tr>
<td>Stop</td>
<td>1.5,1.5</td>
<td>1.5,1.5</td>
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According to forward induction, choosing to play the game is a signal about intended action A.

Q: which action is dominated in player B’s action set?

If ever row player chooses not to stop the game, then she will pick A. Picking B contradicts her not stopping the game.

Then the best option for column player is A (since we assume she is rational enough to realize the above reasoning).

The outside option is NOT played in equilibrium.

This demonstrates the subtle power of outside option (pre-commitment).
This equilibrium refinement is based upon deductive logic.

It might fail if players are not sure about each other’s rationality.

Q: Why we need the following experiment?

Battle of sexes (small outside option)

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<tr>
<td>B</td>
<td>0, 0</td>
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<tr>
<td>Stop</td>
<td>0.5, 0.5</td>
<td>0.5, 0.5</td>
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An algorithm related to forward induction.

One action “strictly dominates” another action if it is superior, no matter what the other players do.
A strictly dominated action should never be used in any Nash Equilibrium.

One action “weakly dominates” another action if both following conditions are satisfied.

(1) it is at least as good as the second, no matter what the other players do;

(2) it is better than the second action for some actions of the other players.

A weakly dominated action may still be used in some Nash Equilibrium.

Dominated Actions

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<td>M</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>1</td>
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</tbody>
</table>
Only row player’s payoff is given.

For row player, B strictly dominates T, M weakly dominates T, and B weakly dominates M.

If people talk, they can agree that strictly dominated action should be deleted from the model (both players understand).

For example,

**Battle of sexes (big outside option)**

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<tbody>
<tr>
<td>A</td>
<td>2, 1</td>
<td>0, 0</td>
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<td>B</td>
<td>0, 0</td>
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<tr>
<td>Stop</td>
<td>1.5, 1.5</td>
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should simply be

**Battle of sexes (big outside option)**

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<td>A</td>
<td>2, 1</td>
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<td>B</td>
<td>0, 0</td>
<td>1, 2</td>
</tr>
<tr>
<td>Stop</td>
<td>1.5, 1.5</td>
<td>1.5, 1.5</td>
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</table>
however, since players move simultaneously without communication, the above model reduction requires the following

(1) the row player is rational

(2) the column player knows (1) (that the row player is rational)

(3) the row player knows (2)

(4) …

this is called common knowledge of rationality

it is stronger than the assumption of rationality, which is (1) above.
we can apply the principle repeatedly

**Iterated elimination of strictly dominated actions**

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<th>L</th>
<th>M</th>
<th>R</th>
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<tbody>
<tr>
<td>T</td>
<td>3, 1</td>
<td>0, 0</td>
<td>−1, 2</td>
</tr>
<tr>
<td>B</td>
<td>0, 0</td>
<td>1, 3</td>
<td>0, 2</td>
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step 1: L for column player is dominated by R and eliminated. The model is reduced to

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<th>M</th>
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<tbody>
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<td>T</td>
<td>0, 0</td>
<td>−1, 2</td>
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<tr>
<td>B</td>
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step 2: T for row player is dominated by B and eliminated. The model is reduced to

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<th>M</th>
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<td>B</td>
<td>1, 3</td>
<td>0, 2</td>
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</table>
step 3: R for column player is dominated by M and eliminated. The model is reduced to a single outcome

\[
\begin{array}{c|c}
\text{B} & \text{M} \\
\hline
\text{B} & 1, 3 \\
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\]

Lecture 4: Collective Decision-Making

This lecture introduces some applications of strategic game in collective decision-making.

We mainly rely upon voting theory to illustrate this point.

Voting theory is an effective tool to model many collective decision-makings.

It explicitly or implicitly models a range of phenomena in social activity, business, and democratic/elite politics.
From the perspective of an abstract social planner who aims to make good collective decision, the important task is to

(1) design an institution which reflects the preference of the majority (or more generally speaking possessing desirable properties such as efficiency and fairness)

(2) make people comply with the institution, eg, by honestly supplying private information regarding preferences. Otherwise people have incentive to manipulate the outcome.

Summarizing, one task is preference aggregation, the other is information aggregation.

Example: a class day trip to suzhou or hangzhou?

We also introduce the concept of dominance which is a useful tool for understanding games.
Using this tool we show that people are honest with their preference in a majoritarian institution.

Majoritarian collective decision-making institutions can be sustained by competition.

4.1 HONESTY

Honesty is not a trivial problem. People might strategically mis-represent themselves.

Example: a class day trip to huang shan, suzhou or hang zhou?

In a majoritarian collective decision-making institution, people can be induced to honestly supply private information regarding preferences, etc.

A very simple model:
Two candidates A, and B, vie for office. Each of an odd number of citizens may vote for either candidates. Abstention is not permitted. The candidate who obtains the most votes wins. We know that a majority of citizens prefer A to win.

Claim: a citizen’s voting for her less preferred candidate is weakly dominated by her voting for her favorite candidate.

In other words, no one has the incentive to lie.

Q: how should we translate the definition of weak domination to our situation?

To prove it, we show that switching either does not affect outcome, or creates a worse outcome.

Q: Is everyone voting B a Nash Equilibrium?
A more general model:

The member of a group of people are affected by a public policy, modeled as a number on the real line.

Each person $i$ has a favorite policy, denoted $x_i^*$.

Restriction upon the preference (single-peaked preference): she prefers a policy $y$ to the policy $z$ if and only if $y$ is closer to $x_i^*$ than is $z$.

The number $n$ of people is odd.

The following mechanism is used to choose a public policy: each person names a policy, and the policy chosen is the median of those named.

For example, if there are 5 people and the policies named are $-2, 0, 0.6, 5, 10$, then the official policy will be $0.6$. 
Does anyone has an incentive to report something other than her preferred policy?

Formally speaking, we have a game

Players: \( n \) people

Actions: each person’s set of actions in the set of real numbers (policies).

Preferences: each person \( i \) prefers the action profile \( a \) to the action profile \( a' \) iff the median policy named in \( a \) is closer to \( x^*_i \) than is the median policy named in \( a' \).

Claim: a citizen’s naming any number other her most-preferred policy \( x^*_i \) is weakly dominated by naming \( x^*_i \).

Proof: We do it in two steps. First we show that for all possible actions of all other players, switching from \( x^*_i \) will never make player \( i \)
strictly better. Second we show that for some actions of other players, switching from $x_i^*$ will make player $i$ strictly worse off.

\[ \forall \text{ action profile } a_{-i}, \text{ denote the value of the } \frac{1}{2}(n-1)th \text{ highest action by } a \text{ and the value of the } \frac{1}{2}(n+1)th \text{ highest action by } \bar{a}. \]

(so that if we order the values of the rest of the players, the first half of the remaining players’ actions are at most $a$, while the last half of the remaining players’ actions are at least $\bar{a}$). Obviously $a \leq \bar{a}$.

For example, if $n = 5$, and policies named by the rest four players are $-2, 0, 5, 10$, then $a = 0$, and $\bar{a} = 5$.

We can illustrate the ideas using a graph.

We have also shown that, everyone naming her favorite policy constitutes a Nash Equilibrium.
It is obviously not the only NE. For example, everyone naming 1 is also a NE.

4.2 HOTELLING MODEL

Suppose information revelation is not a problem, we deal with the preference aggregation problem.

Election competition can be an effective method to make the collective action reflect the preferences of the majority.

Model due to Hotelling (1929). Also a story about street vendor and product differentiation.

The concept of best response.

Suppose the policy space is the real line (e.g. left-right). Think about it as the many cities
from nanjing to hang zhou, all candidates for day trip.

Two candidates are competing for winning an election.

They only cares about winning, not at all about policy per se.

They can announce a number as a proposed policy, which will be automatically implemented if its proposer gets elected.

The voters care only about policy to be implemented.

Their preference is single-peaked. We call the most-preferred point an ideal point.

Additional restriction (only to simplify analysis): their preference is symmetric
(say, quadratic utility function, write $x^*$ as the policy eventually implemented)

$$U = -(x^* - x^*_i)^2$$

Voters vote honestly (they vote for the policy closer to their most-preferred point)! We are assuming it for the moment.

Formally speaking:

Players: two candidates

Actions: real line

Preference: winning outright is better than tie, which is better than losing.

Class experiment: for example, a society of 7 people with ideal points $-1.2, 0, 0.6, 1, 5, 8, 10,$
ask two students to simultaneously submit a policy.

Show that both choosing the median of all the voters is a NE, with the help of sequential-move election (best response functions).

We need to draw two graphs of best response functions. Note that the distribution of ideal points should not matter.

There is only one intersection.

A more direct proof:

(1) it is a NE

(2) There is no other NE. Proof by contradiction. If there is one NE in which one candidate loses, she can move to the median to guarantee at least a tie. If there is one NE as a
tie, then either candidate can win outright by moving to the median.

Some comments:

Competition is not necessarily the only way to achieve majoritarian collective decision-making.

But it is a stable (not unstable) way to do so. Empirical evidences abound.

And as previous section demonstrates, people generally have incentive to report true private data in this game.

The prediction of absolute convergence to the median is of first-order importance.

One of the major applications is the determination of tax rate (Romer 1975).
In this model people do three things: work to earn an income, consume, and participate in a political process (through vote) to determine a proportional income tax rate. All tax will be eventually redistributed equally back to everyone. So people votes to determine the size the redistribution.

we can show that

Everyone more wealthy than the median voter prefers lower tax rate than what is preferred by the median voter. Their preference is stronger the further away their position is from the median.

Everyone less wealthy than the median voter prefers higher tax rate than what is preferred by the median voter. Their preference is stronger the further away their position is from the median.
This simple model predicts that the political equilibrium tax rate is determined by the position of the median voter in the income distribution.

Theoretical predictions

If the median’s income is above average, she prefers negative tax rate.

If the median’s income equals to average, she prefers 0 tax rate.

If the median’s income is below average, she prefers positive tax rate.

In practice, the income distribution has a median income below the mean income. This is due to the nature of right-shewed income distribution we observe everywhere.
Why there are always more poor people than rich people? Labor economists analyzed this phenomena as early as Roy (1951).

So a majority of (poor) voters would always favor redistribution through proportional income taxation.

summarizing, the most important prediction

The greater is income inequality as measured by the distance between median and mean income, the higher the tax rate (or the size of redistribution program, or the pressure to redistribute). If the median voter is relatively worse off with income well below the mean income, then equilibrium redistribution (or the pressure to do so) is large.

Political economists have conducted many empirical research to test the predictions of the theory.
It is indeed true that when western countries democratize (median voter becomes lower and lower), we observe more and more redistribution.

Cross-country comparisons in general have yielded only weak evidences for the theory.

Why: the complexity of the problem and the difficulty in designing an empirical strategy. we focus upon the first here as some discussion of the problem.

Aristotle wrote: “where democracies have no middle class, and the poor are greatly superior in number, trouble ensures, and they are speedily ruined.” So the tendency of the poor to redistribute is well-known. This most likely creates endogenous institutions to check it. For example, the value of property rights in western democracies.
The extent of this redistribution decreases with the elasticity of labor supply.

The real political institution is complex. For example, who can be nominated as candidates affects the election quality.

Politicians have incentive to lie before being elected. And it is difficult to monitor incumbent officials (moral hazard).

Q: For rural election in China, what we learn from the Hotelling model and median voter theorem?

Lecture 5: Public-Good Provision

We now study a model in which two players decide how much to contribute to a public good.

A good is public if it is non-rival and non-excludable.

5.1 FREE-RIDING INCENTIVE

Denote person $i$’s wealth by $w_i$, the amount she contributes to the public good by $c_i$ ($0 \leq c_i \leq w_i$).

The player spends the rest of her wealth on private good (money).

The production function of public good is simply the summation of the contributions.
We assume the utility simply as

\[ u_i(c_1, c_2) = v_i(c_1 + c_2) + (w_i - c_i) \]

or simply

\[ u_i(c_1, c_2) = v_i(c_1 + c_2) - c_i \]

The players derives direct utility from private good (more is better), but for public good, there is an optimal level, less than or larger than this level makes the player worse off. (See Figure 43.1). Technically it is a quasi-concave utility function for public goods.

It is additive in respect to the relationship between private and public goods.
Q: What is the player, action and preference of this game?

We want to take a look at the best response function of this game.

player 1’s best response function

take $c_2 = 0$, we need to find $c_1$ such that $u_1(c_1, 0)$ is maximized.

$b_1(0)$ exists. Let’s first assume $0 \leq b_1(0) \leq w_1$ to get interior solution.

Q: What’s the economic interpretation?

then take any $c_2 = k > 0$, we need to find $c_1$ such that $u_1(c_1, k)$ is maximized.

We need to find a way to connect this to what we have above for $c_2 = 0$. After taking a look
at the functional form we discover that if we can fill the following

$$u_1(c_1, k) = u_1(c_1 + k, 0) + ?$$

these two cost-sharing scheme makes the provision of public goods at the same level. (Q: what does it imply for the cost sharing? )

It is easy to figure out that $? = k$. To check add up the following two equations.

$$v_1(c_1, k) = v_1(c_1 + k, 0)$$

$$-c_1 = -(c_1 + k) + k$$
We we have

\[ u_1(c_1, k) = u_1(c_1 + k, 0) + k \]

It simply means that the curve of the left-hand side is simply a left-upper move of the original \( u_1(c_1, 0) \) curve. And the move is \( k \) on both directions.

Then the best response is \( b_1(k) = b_1(0) - k \).

Q: What’s the economic interpretation?

The same thing happens to player 2, too.

Draw the graph. F44.1

(Two parallel lines, connected on the x axis or y axis. For an interval of contributions of
the other player, one player’s best response is always 0)

Q: then what happens in the NE? can there be multiple NEs?

Q: Is this level of public good provision fair? What do you think is the fair level?

5.2 DISCRETE PUBLIC GOOD

In our previous example the public good is a continuous good.

In general, a continuous public good will be under provided due to people’s incentive of free riding. Players’ dominant strategies are always contributing less.

Q: which of the game we studied before has this feature? what is the public good here?
Sometimes the public-good size is fixed, the only question is whether it should be provided or not (binary choice).

Example, whether to buy a computer for the entire dorm or not. People have agreed upon what computer to purchase, so the price is fixed. But if the contribution is lower than the price, the public good is not provided.

You may still have incentive to free ride your roommates, but not all NEs will fail to provide the public good (corresponding to under-provision in continuous case).

The model:

There are 3 players. There is one unit of public good to be provided. Each person contributes a fixed quantity $c$. And the price of the public good is $2c$. Any contribution will not be refunded if the public good is not provided.
Each player has the same preference.

(1) Public good is provided with no personal contribution.

(2) Public good is provided with her personal contribution.

(3) Public good is not provided with no personal contribution.

(4) Public good is not provided, but she has made personal contribution.

Informal Quiz:

(1) Is it a weakly dominant strategy not to contribute?

(2) Is it NE that exactly 3 players contribute?
(3) Is it NE that exactly 2 players contribute?

(4) Is it NE that exactly 1 players contribute?

(5) Is it NE that exactly 0 players contribute?

(6) Suppose we modify the question and make the total number of players to be only 2, and one unit of public good is provided if only one contributes. Which of the games we studied before most closely models this situation?

(7) Suppose we modify the question and make the total number of players to be only 2, and one unit of public good is provided if both players contribute. Which of the games we studied before models this situation?

Lecture 6: Randomization, Evolution and Correlation

6.1 MIXED-STRATEGY EQUILIBRIUM
Generalizing pure-strategy NE

Definition: mixed strategy $\sigma^i \in \Delta(S^i)$

Strategy space is on probability (example of finite actions)

Payoff is expected

Def: A mixed-strategy Nash Equilibrium specifies a mixed strategy $\sigma^i \in \Delta(S^i)$ for each player $i$ such that the expected payoff of using $\sigma^i$ is no worse than any other mixed strategy $\rho^i \in \Delta(S^i)$, giving $\sigma_{-i}$.

Theorem (Nash): Every n-person game in which each player has finitely many (pure) strategies possesses at least one mixed-strategy Nash equilibrium.

Example: dove-hawk (two animals fighting over a prize)
There are two pure-strategy Nash equilibria of the game: One animal chooses Hawk and the other chooses Dove.

There is also a mixed-strategy equilibrium in which each animal chooses Hawk with probability 1/3 and Dove with probability 2/3.

Best response curve

Example: discrete public good

In our previous discussion an intuitive symmetric equilibrium does not exist for this symmetric game.

Everyone contributing (or everyone not contributing) is not NE.
Theorem: A finite symmetric game has a symmetric mixed-strategy equilibrium. (proof omitted)

This mixed-strategy equilibrium could be more intuitive than other pure-strategy equilibria mentioned above because it is symmetric. It is a natural candidate of equilibrium selection when the players are homogenous, or when players are not sure about any salient labels among each other which might serve as focal point for coordination.

Let’s use the example of reporting a crime to illustrate.

A crime is observed by a group of $n$ people. Each person would like the police to get informed but prefers that someone else make the phone call. Specifically, suppose each person attaches value $v$ to the police being informed
and bears the cost \( c \) is she makes the phone call. We assume \( v > c > 0 \).

Calling the police is a discrete public good for a group of people. It will be provided if and only at least one person contributes. This is a special case of our previous general example \((k = 1)\).

We know that it has \( n \) pure NE in which exactly one player makes the phone call. However who should call represents a serious equilibrium selection problem in a coordination game.

No symmetric pure NE exists. Everyone one calling or No one calling are not NE.

We can check that it has a symmetric mixed-strategy NE in which everyone calls with probability \( p \).
In any such equilibrium, each person’s expected payoff to calling is equal to her expected payoff to not calling.

A person gets $v - c$ for sure if she calls.

If she does not call, a person gets 0 if no one else calls, and she gets $v$ if at least one of the others calls.

Then in equilibrium

$$v - c = 0 \cdot \Pr\{\text{no one else calls}\}$$

$$+ v \cdot \Pr\{\text{at least one other person calls}\}$$

or

$$c/v = \Pr\{\text{no one else calls}\}$$

It simplifies to
\[ p = 1 - \left(\frac{c}{v}\right)^{1/(n-1)}. \]

Observation 1: As \( n \) increases, \( p \) decreases.

This is not very surprising. It follows from people’s incentive to free ride others.

What about the probability that at least one person calls when group sizes increase? Can larger size compensate individual’s reinforced incentive to free ride?

The event “no one calls” is the same as “\( i \) does not call & no one other than \( i \) calls.”

Since people’s choices are independent. Then we have

\[
\Pr\{\text{no one calls}\} = \Pr\{i \text{ does not call}\} \cdot \Pr\{\text{no one else calls}\}
\]
We know the first term on the right-hand side increases as group size increases.

We know the second term on the right-hand side remains constant \((c/v)\) as group size increases.

We conclude

Observation 2: As \(n\) increases, \(\Pr\{\text{no one calls}\}\) also increases.

The larger is the group, the smaller is the probability that the police is informed of the crime (or in general, the public good is provided)!

Is there any difference between America and China concerning the following case? 36 people witnessed the brutal murder of Catherine Genovese in NYC, March 1964, but no one even called the police, not to mention help.
Are people simply more indifferent to each other in big cities?

Other psychological explanations (diffusion of responsibility, audience inhibition, social influence, etc) discuss how people might feel relatively less desirable to intervene with a larger group. It is hard to conduct scientific analysis when preference is so flexible. You can explain anything.

Our equilibrium analysis fixes the cost and benefit of intervention independent of the size of the group. In other words, we fix preference. And we use a very general model (fitting both America and China, and more) to derive some equilibrium implications, which seem to be compatible with the data.

More property of mixed-strategy NE
Proposition: A collection \((\sigma^1, \ldots, \sigma^n)\) of mixed strategies is a Nash equilibrium if and only if for each player \(i\), it is true that \(\sigma^i(s^i) > 0\) implies that \(s^i\) is a best response to \(\sigma_{-i}\).

In words, the proposition says that the equilibrium condition is that for each player \(i\), the mixed strategy \(\sigma^i\) can give positive weight only to pure strategies \(s^i\) that maximize player \(i\)'s expected payoff, given the mixed strategies used by the other players.

We can verify it.

Proof is left as homework

6.2 Some interpretations of mixed strategy

1. One (naive) view is that the player deliberately introduces randomness into his behavior.
That is, a player who uses a mixed strategy commits to a randomization device.

There are certainly cases in which players introduce randomness into their behavior. This happens a lot in competitive situations as sports and war. More examples: R or L in tennis serve; penalty kick in soccer game; players randomly “bluff” in poker, governments randomly audit taxpayers, and some stores randomly offer discounts.

Criticism:

Why the players want to introduce randomness into their behavior?

a) This is usually done in order to influence the behavior of other players.

b) In equilibrium a player is indifferent between his mixed strategy and any other mixture of
the pure strategies with positive probabilities in his equilibrium mixed strategies. (see above proposition)

Apart from this difficulty, there is the feeling that perhaps people do not really randomize when making decisions.

2. Mixed Strategies as beliefs

A mixed strategy NE is a profile $\beta$ of beliefs, in which $\beta_i$ is the common belief of all the other players about player $i$’s actions, with the property that for any $i$ each action used with positive probability in $\beta_i$ is optimal given $\beta_{-i}$ (see above proposition).

Each player chooses a single action rather than a mixed strategy. An equilibrium is a steady state of the players beliefs, not their actions.
These beliefs are required to satisfy two properties: they are common among all players and are consistent with the assumption that every player is an expected utility maximizer.

Criticism:

When we interpret mixed strategy equilibrium in this way the predictive content of an equilibrium is weak.

3. (based upon evolution) Think about the dove-hawk game. Let’s imagine that there is a (large) population of animals and that members of this population encounter one another at random. Mixed strategies can then be thought of as distributions of (pure) strategies in the population.

Thus, the mixed strategy above corresponds to the case in which 1/3 of the population
employs the Hawk strategy while 2/3 of the population employs the Dove strategy.

Moreover, this proportion appears to have a kind of stability property. If the proportion of Hawks were 1/2, say, then Doves would have higher expected payoffs than Hawks, and, for this reason, might be expected to increase in the population. Likewise, a population consisting exclusively of Hawks, or exclusively of Doves, would not be stable.

In other words, a group of all doves is not stable (hawk mutant will survive and increase until 1/3 of the group is hawk)

Similarly a group of all hawks is not stable either.

This interpretation calls for another solution concept, which fits the evolutionary forces in
preventing other steady situations. Such a concept is the *Evolutionary Stable Equilibrium*.

yet another interpretation

4. Mixed Strategies as Pure Strategies in an Extended Game

Under the next interpretation, before a player selects his action he consciously or not receives random private information, inconsequential from the point of view of the players, on which he depends his action.

A mixed strategy NE captures the dependence of behavior on factors that the players perceive as irrelevant. A mixed strategy NE, viewed in this way, is a description of a steady state of the system reflecting elements missing from the model.
In BoS for example, the strategy might reflect a mood of the player (insisting or giving).

Criticisms:

First, it is hard to accept that the deliberate behavior of a player depends on factors that are irrelevant. We usually give reasons for choices.

Second, the behavior predicted by an equilibrium under this interpretation is very fragile. If a manager’s behavior is determined by the type of breakfast he eats, then factors outside the model, such as a change in his diet or the price of eggs, may change the frequency with which he chooses his actions, thus inducing changes in the beliefs of the other players and causing instability.

Finally, in order to interpret an equilibrium of a real life problem in this way one needs to
indicate the exogenous variables on which the players base their behavior. For example, to interpret a mixed strategy NE in a model of price competition one should both specify the unmodeled factors that serve as the basis for the firms pricing policies and show that the information structure is rich enough to span the set of all mixed strategy NE. Those who apply the notion of mixed strategy equilibrium rarely do so.

But, if a mixed strategy NE is a steady state in which each player’s action depends on a signal that he receives from “nature” one might wonder why to exclude equilibria with signals which are not independent.

6.3 Evolution

6.3.1 SYMMETRIC GAME: a group of homogeneous players
The general case of discrete public good game is a symmetric game.

Def: A two-person strategic game is symmetric if every player has the same set of actions, and \( u_1(a_1, a_2) = u_2(a_2, a_1) \).

If you know matrix algebra, it simply says that one player’s payoff matrix is the transpose of the other’s.

In other words, player 1 feels the same about outcome \((a_1, a_2)\), as player 2 feels about the outcome \((a_2, a_1)\). Whether you are row player or column player does not matter (exchangeable).

Prisoners’ Dilemma, Stag Hunt are all symmetric.

BOS, Matching Pennies are not.
Def: a symmetric equilibrium is a NE in which everyone uses the same strategy.

The above 3 symmetric games all have symmetric equilibria.

Symmetric equilibrium is intuitive in a symmetric game.

A homogenous group whose participants all use the same optimal strategy when they are paired to play a game.

However, it is not true that symmetric game always has symmetric pure strategy equilibrium.

For example, an anti-coordination game.

Anti-coordination Game
For a general discrete public good provision game,

There are $n$ players. There is one unit of public good to be provided. Each person contributes a fixed quantity $c$ (the cost of group participation). And the price of the public good is $kc$, with $1 \leq k \leq n$.

Theorem: A symmetric game with two strategies has an equilibrium in pure strategies. (proof omitted)

So we know that $\forall k$: the above discrete public good game has pure-strategy equilibrium.

Using previous knowledge we know one class of equilibria has exactly $k$ players contributing.
If player $i$ is sure that $k - 1$ players have contributed, conditional upon this, she is playing a chicken game with the rest $n - k$ players. This has a flavor of BoS/anti-coordination. And the player wants to free ride if other people contribute.

If player $i$ is sure that $n - k$ players will not contribute, conditional upon this, she is playing a (generalized) stag-hunt game with the rest $k - 1$ players. And the player wants to cooperate, but worries about the risk that others might not cooperate.

This might better explain people’s behavior of voluntary contribution than using Prisoners’ Dilemma game.

So in general discrete public good provision contains both incentives of free-riding and cooperation.
Q: what about $k = 1$?

Q: what about $k = n$?

However, it could be any $k$ players out of the entire $n$ players. This is a large class of potential equilibria.

Which of these $k$ people should contribute? Or in other words, which equilibrium should be selected? This could be a difficult coordination task as $1 < k < n$, especially when the number of combinations big.

If these people are heterogenous, it is possible that players can coordinate using some external labels, like focal point.

For example, when $n = 3$ and $k = 2$, and there are two rich people and one poor people, it is
likely that the public good will be provided by these two rich people.

In general, efficient and fair allocation of the costs can not be easily achieved using the above method.

What if there is one rich people only?

The signal might be weaker if we change richness to gender or race.

Also these signals might contradict each other. For example, what if there are two poor males with one rich female?

*Notes on Equilibria in Symmetric Games* Shih-Fen Cheng, Daniel M. Reeves, Yevgeniy Vorobeychik, Michael P. Wellman

6.3.2 Evolution
We consider a very large group of homogeneous players/organisms whose members are repeatedly and randomly matched in pairs to interact with each other. The outcome of the interaction determines their “fitness.”

For example, a population of identical animals, pairs of which are periodically engaged in conflicts over prey.

A steady state (another way of describing an equilibrium in dynamic situations) is reached if all animals use the same strategy and no mutant can invade this population.

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If every organism takes action A, than if the mutant population $\epsilon$ is below 2/3, the mutant
has lower expected payoff and its population shrinks to 0.

What about everyone taking action B?

\((\epsilon < 1/3)\)

In a sense the action A is more stable because the threshold is higher.

Here the point is that for positive threshold no mutant, starting with a very very small quantity, can invade the population.

In the following we will refer to situation where in each “match” between two players each of the players have to choose from a set \(B\) of “modes of behavior”. The organisms do not consciously choose actions; they either inherit modes of behavior from their forebears or are assigned them by mutation.
The function $u(a, b)$ is the payoff of a player who takes the action $a$ against a player who takes the action $b$. The function $u$ has a new meaning: it measures each organism’s ability (fitness) to survive: if an organism takes the action $a$ and faces the distribution $\beta$ of opponents’ actions, then its ability to survive is assumed to be the expectation of $u(a, b)$ under $\beta$.

A candidate for an evolutionary equilibrium is an action in $B$. An equilibrium is a steady state in which all organisms take this action and no mutant can invade the population.

More precisely, $\forall b \in B$ we view the evolutionary process occasionally transforming a small fraction of the population into mutants who follow $b$. In an equilibrium any such mutant must obtain an expected payoff strictly lower
than that of the equilibrium action, so that it dies out.

Now, if the fraction $\epsilon > 0$ of the population consists of mutants taking the action $b$ while all other organisms take the action $a^*$, then the average payoff of a mutant is $(1 - \epsilon)u(b, a^*) + \epsilon u(b, b)$ while the average payoff of a non-mutant is $(1 - \epsilon)u(a^*, a^*) + \epsilon u(a^*, b)$.

Therefore for $a^*$ to be an evolutionary equilibrium we require

$$(1 - \epsilon)u(b, a^*) + \epsilon u(b, b) < (1 - \epsilon)u(a^*, a^*) + \epsilon u(a^*, b)$$

for all $\epsilon$ sufficiently small (more formally, there exists $\delta > 0$ such that for all $\epsilon \in (0, \delta)$ the inequality is true.
Def: An evolutionarily stable strategy (ESS) of a symmetric game is an action $a^* \in B$ for which the above inequality is satisfied.

We now know that for our previous example, both A and B are evolutionarily stable, even though B action yields low payoff. For example, we call lack of coordination in economic development as a poverty trap. And such a trap is even evolutionarily stable (consider mutants as reformers/heroes who chooses A).

Above definition of ESS is not convenient, because we need to check threshold for each action. To improve, we need to find an equivalent definition which is also easier to use.

I first claim that

“If there exists $\delta > 0$ such that the above inequality is true for $a^*$, then $(a^*, a^*)$ is a Nash equilibrium.”
proof: suppose not, there exists $b$ such that $u(b, a^*) > u(a^*, a^*)$. then the above inequality is violated for $\epsilon = 0$. obviously for very small $\epsilon$ it is also violated (for any $\delta$ you can find such small $\epsilon$). so you can’t find a $\delta$ such that the above inequality is true (a contradiction).

I now claim that for $(a^*, a^*)$ as a strict NE, we can find a $\delta > 0$ such that the above inequality is true.

The reason is that if $u(b, a^*) < u(a^*, a^*)$ for all $b$, it is true for $\epsilon = 0$, then it must also be true for $\epsilon$ small enough.

So ESS is almost NE. but how about non-strict NE? Then there is $b \neq a^*$ as a best response to $a^*$, i.e. $u(b, a^*) = u(a^*, a^*)$. The original inequality is reduced to $u(a^*, b) > u(b, b)$.

Summarizing, the old definition is equivalent to the following new definition:
(i) \((a^*, a^*)\) as a NE

and (ii) for every \(b \neq a^*\) which is also a best response to \(a^*\), \(u(a^*, b) > u(b, b)\).

Intuitively, the first says that when matched with a normal organism, the original strategy is strictly better than a mutant; and the second says that for those mutant who does equally well in (i), when matched with another mutant, it must do strictly worse.

We can check this new definition with our previous example.

So ESS is a subset of NE.

In the above definition we only considered pure strategy, we can similarly define it for mixed-strategy. 2 interpretations: animals also play mixed strategy (wasp nesting behavior: build
own nest or invade an existing nest?); or fractions of animals play pure strategies. We have checked this for the hawk-dove game.

application: lizards

first consider the following well known game

rock paper scissors

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Male side-blotched lizards follow one of 3 reproductive strategies in California.

Lizards with orange throats are very aggressive and defend large territories. (rock)
Those with blue throats are less aggressive and defend smaller territories. (scissors)

Those with yellow stripes on their throats do not defend any territories and depend on their similarity in coloration to females to sneak unnoticed into another male’s territory to mate. (paper)

Paper beats rock because of the size of the territory, but loses to scissors.

The only mixed-strategy NE is taking each action with 1/3 probability ($\alpha^* = (1/3, 1/3, 1/3)$).

But this is not a ESS.

Note that a mutant taking pure strategy A is a best response to normal organism $\alpha^*$, and the expected payoff of mutant interacting with
mutant is $\gamma$, higher than the expected payoff of mutant interacting with non-mutant $\gamma/3$.

This suggests that the lizards composition is unstable (not $1/3$, $1/3$, $1/3$ for each color), exact opposite to our hawk-dove case!

Data shows that in 1990 and 1991 blue throated males predominate; in 1992 orange throated males reached a peak; while in 1993 and 1994 yellow dominates the group. in 1995 the composition returns to the pattern in 1990.


6.4 Correlated Equilibrium

Extension of mixed strategy: correlated equilibrium (due to Robert Aumann, 2005 Nobel Prize winner in Economics)
Battle of the Sexes

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payoff for three NE is (2, 1), (1, 2), (2/3, 2/3), the last one is expected payoff under mixed-strategy NE.

the last one is more fair

Now, consider a situation where a “trusted” authority (not an interested player) flips a fair coin and based on the outcome of the coin toss, recommends the players what they should do.

Say, if the coin shows heads, row player is told to choose A and column player is told to
choose A; if the coin shows tails, row player is told to choose B and column player is told to choose B.

It is important to note that no individual party has an incentive to deviate from what they are told to do.

the expected payoff is now \((3/2, 3/2)\).

A more interesting example of correlated equilibria is the game of Chicken.

This is again a two player game with the payoff matrix as shown below. In this case, the worst outcome occurs when both players dare (D,D). The deterministic Nashes are (D,C) and (C,D) with rewards (7,2) and (2,7), respectively. A randomized Nash has row players and column player choosing C and D with probabilities 2/3
and 1/3, respectively. This randomized Nash has an expected reward of $4\frac{2}{3}$ for each player.

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Now, let’s look what happens in the case of a correlated equilibrium. As before, a trusted party tells each player what to do based on the outcome of the following experiment: Outcome Probability

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<tr>
<td>(D, C)</td>
<td>1/3</td>
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<tr>
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We note once again that the trusted party only tells each player what he/she is supposed to do. The trusted party does not reveal what
the other player is supposed to do. It is a correlated equilibrium if no player wants to deviate from the trusted party’s instruction.

So, in the Chicken example, if the trusted party tells column player to dare (D), then column has no incentive to deviate. This is because column knows that the outcome must have been (C,D) and that row player will obey the instruction to chicken (C).

Next, let us consider the case when column player is told to chicken. Then column player knows that the outcome must have been either (D,C) or (C,C), each happening with equal probability (why equal?)

Column’s expected payoff on playing C conditioned on the fact that s/he is told to chicken is $\frac{1}{2} \times 6 + \frac{1}{2} \times 2 = 4$. 
In the above expression, 6 is the payoff from row also playing C, i.e., the outcome was (C,C) and 2 is the payoff column gets when row plays D, i.e., outcome was (D,C).

If player column decides to deviate, i.e., play D when told to play C, then the expected payoff is lower. (why?)

Since the game is symmetric, row player also has no incentive to deviate from the instruction of the trusted party.

Note that in the case of the correlated equilibrium, the expected reward for each player is 5. This is higher than the expected reward of the randomized Nash.

The existence of a trusted third party (first justified as sun spots) has more natural explanations.
We also require players to do Bayesian updating.


Lecture 7: Subgame Perfect Equilibrium

So far we have been considering only simultaneous game.

Players choose once for all. No timing.

7.1 BASICS OF EXTENSIVE GAME

Describe an abstract example:

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Do a class experiment.

Discuss the results of experiments, and the situation.

**Entry game**

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<th>Fight</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>In</strong></td>
<td>2,1</td>
<td>0,0</td>
</tr>
<tr>
<td><strong>Out</strong></td>
<td>1,2</td>
<td>1,2</td>
</tr>
</tbody>
</table>

Take entry game as an example.

The only thing we need is both parties’ commitment to such a plan.

Before this class we treat action and strategy interchangeably, it is ok for simultaneous game, but from now on we will make distinctions.
Now we can solve the NE of the entry game, treating it just as a simultaneous game, even though in reality people move sequentially.

Entry game

<table>
<thead>
<tr>
<th></th>
<th>Acquiesce</th>
<th>Fight</th>
</tr>
</thead>
<tbody>
<tr>
<td>In</td>
<td>2, 1</td>
<td>0, 0</td>
</tr>
<tr>
<td>Out</td>
<td>1, 2</td>
<td>1, 2</td>
</tr>
</tbody>
</table>

You can see that there are two pure-strategy NEs.

NE is a steady state (stable norm, rematch for a set of large groups of identical players) in which strategy is optimal given other people’s strategies.

In other words, NE can be justified by experiences in playing the game.

But how about (out, fight)?
It seems to be a stable situation for the long run.

Why we feel uncomfortable about this outcome?

Threat is not credible.

Formally we can write a tree.

Extensive form (or simply a Tree) is another way to describe games in which timing is important.

Basic elements of a tree:

a set of players;

a set of sequences (terminal histories): you can't find a longer sequence which contains
this one. Here is (In, Acquiesce), (In, Fight), and Out

player function: it tells who should move after any non-terminal history. $P(\Phi) = ?, P(In) = ?$

preferences, defined over the terminal histories.

Def: a game has finite horizon if its longest terminal history is finite.

Def: finite extensive game has both finite horizon and finitely many terminal histories.

Def: A player’s strategy specifies the action the player chooses for every history after which it is her turn to move.

In other words, it is a complete plan of actions (for all situations, jin nang miao ji).
This might give you the illusion that the meaning of strategy is the same as what we use in common language.

But how about a game in which player 1 moves before and after player 2?

A strategy specifies an action, which according to the plan, will never be reached.

But this is necessary for player 2 to form a belief and play the game.

Did we see something similar before?

You can use a computer to do the real move for you.

Only delegating to a computer is credible!
We already see that the entry game can be described by both strategic and extensive forms.

Now with our (improved) understanding of strategy, we can see why simultaneous game is not that naive, if we take any action in that game to be a complete plan for all contingencies (i.e. a strategy).

7.2 SPE AND BACKWARD INDUCTION

The commitment to Fight is reasonable in strategic formulation.

We need to eliminate incredible commitment in game trees.

Subgame is simply a (whole) branch of the original tree.
Def: A subgame perfect equilibrium is a strategy profile \( s^* \) with the property that in no subgame can any player \( i \) do better by choosing a strategy different from \( s^*_i \), given that every other player \( j \) adheres to \( s^*_j \).

SPE refines NE, so the set of SPE is a subset of the set of NE.

take a look at the entry game

We can imagine a perturbed (as a result of mistake or experimentation) steady state to appreciate SPE.

Kuhn’s Theorem: Every finite extensive game with perfect information has a SPE.

We can use backward induction to prove the theorem (and more importantly, to find SPE in such games).
Chess is solvable.

dictator, ultimatum and trust game

Lecture 8: Asymmetric Information

8.0 BAYESIAN GAME

Review of Bayesian probability theory: $\Omega$, prior, and posterior.

$\Omega = \{H, T\}$ with uniform prior

$\Omega = \{\omega_1, \omega_2, \omega_3\}$ with uniform prior

$\tau(\omega_1) = \tau(\omega_2) = \text{red signal}$

$\tau(\omega_3) = \text{green signal}$

posterior ?
It offers a way to describe difference in knowledge – a person before updating has less information compared to after updating.

To describe diff in knowledge between two persons at the same time can not directly use this framework.

A smart way, by Harsanyi, is to “pretend” that two persons (with diff in knowledge) initially has the same prior, and then one of them obtains additional information to form more informative prior.

Two firms produce good at constant unit cost. They compete with quantity choice. Price determined by the market.

Firm 1’s cost is open information.
Firm 2’s cost is private information. $c_L$ with probability $\theta$, and $c_H$ with probability $1 - \theta$. $c_L < c_H$. It is called common prior belief.

how to understand? pretend that day 1 they have the common prior belief. then day 2 manager of firm 2 acquires her firm’s accounting report, which follows

$\tau(H) = \text{red signal}$

$\tau(L) = \text{green signal}$

then firm 2 manager updates her belief, which leads to diff in knowledge between firm 1 and firm 2 about firm 2’s cost.

A Bayesian game is defined, using this Cournot game.

- Players (finite). 1 and 2.
• Actions. nonnegative numbers.

• Preferences. For action pair \((q_1, q_2)\), state \(i\) (\(L\) or \(H\)), firm 1’s profit function \(q_1(P(q_1 + q_2) - c)\), and firm 2’s profit function \(q_2(P(q_1 + q_2) - c_i)\).

• States (finite). \(\{L, H\}\) (complete description of possible outcomes)

• Signals. Firm 1’s signal function \(\tau_1(H) = \tau_1(L)\). Firm 2’s signal function \(\tau_2(H) \neq \tau_2(L)\). (function is a map, it describes a relationship, not simply a result. also you should not interpret the signalling process literally. it’s like the ancillary line in geometry proof.)
• Beliefs. Firm 1 keeps prior belief. Firm 2 updates with Bayes’ rule and gets perfect information.

Draw a graph of the information structure.

Note: A Bayesian game is an extended game of the original complete-information game.

It is still a strategic game.

• players \((i, \tau_i)\), with \(\tau_i\) conveniently interpreted as types

• actions for \((i, \tau_i)\) is still the actions for \(i\)

• preference: for player \(i\), average over the actions of other players’ types. but never
hinges upon the actions of any other types of player $i$.

So NE is still the same NE.

DEF: A NE of the Bayesian game in this example is a triple $(q_1^*, q_L^*, q_H^*)$, which maximizes the (expected) profit of a particular type or player (if there is only 1 type) given all other types’ equilibrium actions.

We use best response functions to calculate NE.

the L type of firm 2 solves

$$\max_{q_L} [(P(q_1 + q_L) - c_L)q_L]$$

the H type of firm 2 solves
f \[ \max_{q_H} [(P(q_1 + q_H) - c_H)q_H] \]

firm 1 solves

\[ \max_{q_1} [\theta(P(q_1 + q_L) - c)q_1 + (1 - \theta)(P(q_1 + q_H) - c)q_1] \]

Bayesian game is due to Harsanyi (67/68). He assumes common belief and that all difference in the players' knowledge comes from an objective mechanism which randomly assigns information to players.

Note that it is not due to different initial belief.

The formulation about uncertainty is restrictive.
Aumann “rational agents can’t agree to disagree”, application in stock market.

Common belief implies that after a pair of players receive their signals it can not be commonly knowledge between them that player 1 believes that the prob that the state is in some given set to be $x$ and that player 2 believes that this prob is different, though it is possible that their beliefs differ and one of them unsure about other’s belief.

The formulation about uncertainty is also very general!

Bayesian game can describe situations in which players’ characteristics, such as preferences, is not common information; it can also describe uncertainty over belief.

$$N = \{1, 2\}$$
\[ \Omega = \{ \omega_1, \omega_2, \omega_3 \} \]

prior is uniform distribution

\begin{align*}
\tau_1(\omega_1) &= \tau_1(\omega_2) = t'_1, \quad \tau_1(\omega_3) = t''_1 \\
\tau_2(\omega_1) &= t'_2, \quad \tau_2(\omega_2) = \tau_2(\omega_3) = t''_2
\end{align*}

Draw graphs

preference of player 1: b better than c for \( \omega_1 \) and \( \omega_2 \), but reverse for \( \omega_3 \); player 2 is indifferent for all outcomes.

does player 2 know that player 1 prefers b to c in state \( \omega_1 \)?

does player 2 know that player 1 prefers b to c in state \( \omega_2 \)?
does player 1 know player 2’s belief about her preference in state $\omega_1$?

The formulation about uncertainty is general enough to the applied in many places.

Example: for above Cournot game we can model firm 2 as unsure whether firm 1 knows her cost or not (although in fact it does not).

Example: second-price auction. final bidders and final possible valuations. (say, 3 bidders and 3 possible valuations L, M, H)

How to describe it? Prove that in the Bayesian game there is a NE in which everyone bids her valuation honestly.

Example: providing discrete public good. Everyone knows her own valuation but not others’ valuation. The cost of unit public good is common information.
8.1 ADDING UNCERTAINTY TO A TREE

Example: Chain-store Game (a sequence of entry game)

NE? How good is it?

SPE?

How good is it when the number of chain store is small?

How good is it for a large one?

the logic of induction.

But maybe an early fighting incumbent is trying to convey something, which the later challengers should learn!
One way out: introducing uncertainty about the resoluteness of the incumbent, which can be conveyed as the game proceeds.

In Bayesian game people do not learn from history.

To analyze uncertainty in a game tree we need to add little to specifications of a game, but we have to make substantial changes to the solution concepts.

Or more precisely, we have to think more clearly about how people evaluate the uncertainty (i.e. belief).

The new concept is called weak perfect Bayesian equilibrium (weak PBE), or weak sequential equilibrium.

So far players know their positions on the tree.
It is best described by a history.

Any kind of uncertainty eventually can be understood as uncertainty over history. For example, number of players, who played, what actions taken, preferences, etc.

history is but a state, using our notation in Bayesian game. So our formulation about uncertainty is indeed very general.

Of these uncertainties, we distinguish between exogenous ones (specifications) and endogenous ones (actions).

We model the first one as Chance (or Nature) move, then there is no difference between these two.

Define $H_i$ the set of histories after which player $i$ moves.
Suppose, the player $i$ moves after the histories $C, D, E$ (i.e. $H_i = \{C, D, E\}$).

If the history is $C$, she knows it.

If the history is $D$ or $E$, she only know that one of them has occurred, but not which one.

Her belief can be represented probabilistically.

We describe player $i$’s information partition as consisting of two information sets $\{C\}$ and $\{D, E\}$

What is the most informative partition for player $i$?

The concept of information set generalizes the concept of node in the game tree.
Example: entry game (challenger might be prepared or not)

Entry game II

<table>
<thead>
<tr>
<th></th>
<th>Acquiesce</th>
<th>Fight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ready</td>
<td>3, 3</td>
<td>1, 1</td>
</tr>
<tr>
<td>Unready</td>
<td>4, 3</td>
<td>0, 2</td>
</tr>
<tr>
<td>Out</td>
<td>2, 4</td>
<td>2, 4</td>
</tr>
</tbody>
</table>

what's the information set?

(every player only has one information set)

8.2 Motivations for weak PBE

Recall the definition of strategy for perfect-information extensive game as

Def: A player’s strategy specifies the action the player chooses for every history after which it is her turn to move.
Now it generalizes to

Def: A player’s strategy specifies the action the player chooses for every information set after which it is her turn to move.

We have shown that the notion of NE is not adequate in all extensive games with perfect information.

And we developed the notion of SPE to handle this problem.

Everything we said before is still true for the imperfect-information extensive game, because perfect-information extensive game is a special case of it.

Consider entry game II.

Again there are two pure-strategy NE.
Again (Out, Fight) is not a reasonable NE, why?

The natural thing to do is to require that each player’s strategy be optimal at each information set.

In entry game II, this could be done, because it is clearly suboptimal to fight.

But what if the incumbent prefers to fight than to accommodate an unprepared entrant?

Again there are two pure-strategy NE.

Given that fighting is now optimal if the challenger enters unprepared, To see whether (Out, Fight) is reasonable NE or not depends upon the incumbent’s belief of the history.

The challenger’s strategy Out gives the incumbent no basis upon which to form such a belief.
So it is not obvious how to apply the notion of SPE in our more general setup.

We have to explore ways to define belief.

We know NE of a strategic game may be characterized by two requirements:

i) that each player choose her best action given her belief about other players

ii) and that each player's belief be correct (based upon other player’s choices)

There was no need for us to separately define belief.

But now it has to be done, just as the above example has shown.
And the basic requirement for such a belief is still the same as that for a strategic game, i.e., the belief should be consistent with other player’s strategies (i above).

And like the notion of SPE for perfect-information extensive game, we also require that the players are rational at any point (here is any information set) of the game.

8.3 WEAK PBE

Def: a belief system is a collection of beliefs, one for each information set of every player.

Def: An assessment in an extensive game is a pair consisting of a profile of strategies and a belief system.

An assessment is an equilibrium if it satisfies the following two requirements:
Sequential rationality: each player’s strategy is optimal whenever she has to move, given her belief and the other players’ strategies.

Consistency of beliefs with strategies: each player’s beliefs is consistent with the strategy profile.

The assessment makes belief explicit in our solution concept.

You can do it to analyze NE and SPE too.

Example: BoS

Example: Entry game II, (Ready, Acquiesce) is weak PBE.

What about (Out, Fight) in entry game II?
The information set of the incumbent is not reached in equilibrium, and we place no restriction upon the incumbent’s belief.

We eliminate this equilibrium any way, just as in a SPE, because it is not a rational choice to fight for any belief.

For this reason we can’t eliminate this equilibrium if the incumbent prefers to fight only if the challenger is not ready.

What if the information set of the incumbent is reached as a result of challenger’s strategy?

For example, denote by $p_R$, $p_U$, and $p_O$ the probabilities the challenger assigns to the three actions. They have add up to 1, obviously. Then what should be the consistent belief for $p_O < 1$?
Formally speaking, we require that the probability assigned to every history $h^*$ in an information set by the belief of the player who moves after there to be equal to the actual probability that $h^*$ occurs according to the strategy profile, conditional upon the information set’s being reached.

Denote by $I_i$ the concerned information set and for the given strategy profile, Bayes’ Rule applies and we have the belief for $h^*$ to be the probability

$$\frac{\Pr(h^*)}{\sum_{h \in I_i} \Pr(h)}$$

Note that weak PBE still leaves a lot of equilibria, some of which probably questionable using more restrictive solution concepts. But we can go a long (really long) way just using this one.

Lecture 9: Signalling
A (very large) class of economic and political problems can be understood as signalling game.

Some player is better informed of certain aspects of the game.

So uncertainty is exogenous, and Chance moves first.

Some players know Chance’s moves, but not others.

9.1 EDU AS SIGNAL OF ABILITY

Motivations: ability is valuable on the labor market, but unobservable. Even if we assume that everyone know his/her true ability, how can the firms give a job to anyone who claims to be able?
One way out: the mundane role of education

H, L types of worker(s).

You can think of a type as a rational agent.

Two firms simultaneously offer to the worker.

Worker effort cost $e/L$ or $e/K$ for L and H, respectively.

Note that $e/L > e/K$.

Productivity equals to ability.

Game Tree of the problem

Let’s check the following weak PBE.

Worker: Type H choose $e = e^* > 0$, and type L choose $e = 0$; after observing the firms’ wage
offers, both types choose the highest offer if they differ, and that of firm 1 if they are the same.

Firms’ belief: H if observing $e^*$, L otherwise.

Firms’ strategies: offers H to high type and L to low one.

$$L(H - L) \leq e^* \leq H(H - L)$$

pooling and semi-pooling equilibrium

Exercise 342.1

9.2 STRATEGIC INFORMATION TRANSMISSION

The cost was crucial in the above model to signal your information.
In other words, it was not enough to simply talk.

One motivating example: principal/agent, Sender-Receiver Game by Crawford & Sobel (1982)

1. 2 players: sender, receiver

2. state of world $\omega \sim \text{unif}[0, 1]$ 

3. The sender knows exact $\omega$.

4. The receiver only knows the probability distribution.

Sequence of the game:
1. the sender sends a costless and non-verifiable signal to the policy-maker

2. the receiver chooses the policy $y$ and both players get the payoffs

Payoff functions

\[
U^S(y, \omega, b) = - (y - (\omega + b))^2 \quad (1)
\]
\[
U^R(y, \omega) = - (y - \omega)^2
\]

At state $\omega$,

- the receiver’s optimal policy is $\omega$
- the sender’s optimal policy is $\omega + b$
- $b$ measures the fixed bias between these two players
Truth telling (complete separating equilibrium) is not an equilibrium.

Ignoring all information (complete pooling equilibrium) is always an equilibrium.

More interestingly, Crawford & Sobel (82) identified semi-pooling equilibrium.

Also called partition equilibria for this game.

An example of 2-partition equilibrium.

- 2 signals sent in equilibrium, corresponding to 2 policy outcomes.

- For $\omega \in [0, \omega^*]$, signal 1 is sent, with $y^* = \frac{\omega^*}{2}$ implemented.
• For $\omega \in [\omega^*, 1]$, signal 2 is sent, with $y^* = \frac{1+\omega^*}{2}$ implemented.

• Given sender's strategy, receiver maximizes her own expected utility.

• Given receiver’s strategy, for any $\omega$, sender will not send the false signal.

• The smaller the bias $b$, more signals can be sent, information market is more efficient.

• On the other hand, bigger $b$ is less efficient.

• For bias big enough ($b \geq \frac{1}{4}$), only one signal is sent, information market breaks down.
The choice of optimal delegation rule.

Lecture 10: The Core

Example: dividing a piece of gold among three pirates

class experiment

How to model it? What's the stability criteria here?

10.1 BASICS OF COALITIONAL GAME

Characteristics: game rule not completely specified, coalition is possible

Def: A coalitional game consists of

- a set of players
• for each coalition, a set of actions

• for each player, preferences over the set of all actions of all coalitions of which she is a member

Notation: grand coalition as \( N \), an arbitrary coalition as \( S \).

The actions available to a coalition might have nothing to do with the actions available to a member of it.

Def: transferrable payoff

Def: we refer to the total payoff of a coalition \( S \) in a game with transferable payoff as the *worth* of \( S \), and denote it as \( v(S) \).
A coalitional game with transferable payoff is our object of analysis.

It can be completely specified by $N$ and $v(S)$.

example: 3-player majority game

example: 2-player unanimity game

example: one landowner, $m$ worker, with output $f(k+1)$, with $0 \leq k \leq m$.

Our solution concept is the Core.

Def: the core of a coalitional game is the set of actions $a_N$ of the grand coalition $N$ such that no coalition has an action that all its members prefer to $a_N$.

In words, no improvement is possible for any group of players.
For the special case of transferrable utility, can we define the core in worth function?

Note the core always exists since it is a set, and a set technically contains at least the null.

But a core can only contain the null, in that case we say it's empty.

Consider the above examples especially the last one, suppose the size of $N$ is $n$. Allocation $x_1, x_2, x_3$ is in the core if and only if

\[
\begin{align*}
x_1 & \geq f(1) \\
x_2 & \geq 0 \\
x_3 & \geq 0 \\
x_1 + x_2 & \geq f(2) \\
x_1 + x_3 & \geq f(2) \\
x_1 + x_2 + x_3 & = f(3)
\end{align*}
\]
Use the fact that $x_1 = f(3) - x_2 - x_3$ to simplify

\[
0 \leq x_2 \leq f(3) - f(2) \quad (3)
\]
\[
0 \leq x_3 \leq f(3) - f(2)
\]
\[
x_2 + x_3 \leq f(3) - f(1)
\]
\[
x_1 + x_2 + x_3 = f(3)
\]

Economic meaning?

Suppose marginal productivity is decreasing, we can get rid of the third requirement.

The rest are conditions for the core.

\[
f(3) - f(1) \geq 2(f(3) - f(2)) \quad (4)
\]
\[
\geq x_2 + x_3
\]
Economic meaning?

Homework: prove the same conditions for core hold for any $n$. Hint: $f(n) - f(k) \geq (n - k)(f(n) - f(n - 1))$.

(optional homework) also prove the mathematical hint.

How can unions help?

Not to join deviating coalitions except as a union.

10.2 Two-sided Matching

woman-man, student-college, worker-firm, etc.

one-to-one matching is simple, and its results can be generalized to more complex situations.
What is the stable result?

Notation: Two sides are described as $X$s and $Y$s. Any player $i$ in matching $\mu$ has partner $\mu(i)$.

You can think about the function of $\mu$ as the marriage certificate from which you can find the name of any married person's spouse.

Each person only cares about their own partner.

For convenience of analysis preference is strict.

Def: A matching is in the core iff

1. each player prefers her partner to being single
2. for no pair \((i, j)\) consisting of an \(X\) and a \(Y\) is it the case that \(i\) prefers \(j\) to \(\mu(i)\) and \(j\) prefers \(i\) to \(\mu(j)\)

In other words, the second case says that we can restrict our attention to a deviating (new) pair.

Is it a valid definition?

Proof: Easy to show necessity. For sufficiency, proof by contradiction. Suppose there is \(\mu\) not in the core, then ... either one of the above conditions will be violated.

But how can we find the core?

An hypothetical algorithm. Class experiment of 6 speaking computers for the following table.
Deferred acceptance procedure with proposals by Xs.

\[
\begin{array}{ccc|ccc}
  x_1 & x_2 & x_3 & y_1 & y_2 & y_3 \\
y_2 & y_1 & y_1 & x_1 & x_2 & x_1 \\
y_1 & y_2 & y_2 & x_3 & x_1 & x_3 \\
y_3 & & & x_2 & x_3 & x_2 \\
\end{array}
\]

Is there room of improvement for any human computer?

The above procedure might make a difference if offers are made by Ys.

Using only 4 human computers for a simpler game.

\[
\begin{array}{cc|cc}
  x_1 & x_2 & y_1 & y_2 \\
y_2 & y_1 & x_1 & x_2 \\
y_1 & y_2 & x_2 & x_1 \\
\end{array}
\]

And the power to make offers is valuable, though in reality this interpretation should be taken
with a bit caution. Other reasons might affect why males propose in our dating market, and why students apply for jobs after they graduate, etc.

Applications: Medical school grads matched to hospitals in very short time, a response to market unraveling when hospitals competed to maker early offers.

Literatures: A similar algorithm of doctor-hospital matching was used without knowing early game theoretic works in the field (Gale and Shapley 1962). In the 1990s Al Roth was hired to design a better and fairer matching market.

Lecture 11: Persuasion

Understanding persuasion could be extremely important. But ...
It could be communicated through actions, words, texts, eye contact, . . .

The art of persuasion is situational.

Game theorists are interested in studying regularity of persuasion.

11.1 EQUILIBRIUM ANALYSIS

NE: consistency requirement upon rationality and belief.

No one is cheated in a (persuasion) equilibrium.

Example: worker knows more (better)

<table>
<thead>
<tr>
<th></th>
<th>hard</th>
<th>easy</th>
</tr>
</thead>
<tbody>
<tr>
<td>high</td>
<td>2, 1</td>
<td>0, 0</td>
</tr>
<tr>
<td>low</td>
<td>0, 0</td>
<td>1, 3</td>
</tr>
</tbody>
</table>
Consider \((H, L)\)

When interests coincide, persuasion could ...

What matters is that,

- Will the worker lie?
- Is the firm cheated?

Or formally

- the worker has no incentive to send different signal
- the firm’s belief is consistent with the worker’s incentive
This becomes a Nash equilibrium (also PBE).

Even though \((H, L)\) is “natural,” anything could emerge here.

Consider \((L, H)\), (Changjiang, Huanghe).

Even the number of signals could be bigger than 2.

\((H_1, H_2, L)\)

Strategic v.s. natural meaning of a signal

All coding systems (including languages) are equivalent in this sense.

Obviously it has to be shared.

Accordingly, effective persuasion should fail in the following
<table>
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</tr>
</thead>
<tbody>
<tr>
<td>high</td>
<td>2,1</td>
<td>0,0</td>
</tr>
<tr>
<td>low</td>
<td>2,0</td>
<td>1,3</td>
</tr>
</tbody>
</table>

Interests still coincide a bit, but not completely.

default job is hard . . .

default is easy . . .

The number of effective signal?

pooling v.s. separating equilibrium

Note that I did not specify the right choice for the firm.

11.2 DISCUSSIONS

What about advising instead of persuasion for both examples?
What about persuading people to do something with you?

Suppose you have made up the mind, this becomes the same problem.

eamples: a team of worker 1 & worker 2

Additional thing to check: incentive not to switch

<table>
<thead>
<tr>
<th></th>
<th>high</th>
<th>low</th>
</tr>
</thead>
<tbody>
<tr>
<td>high</td>
<td>2,1</td>
<td>0,0</td>
</tr>
<tr>
<td>low</td>
<td>0,0</td>
<td>1,3</td>
</tr>
</tbody>
</table>


one interesting example (Aumann 1990):
<table>
<thead>
<tr>
<th>high</th>
<th>low</th>
</tr>
</thead>
<tbody>
<tr>
<td>high</td>
<td>3,4</td>
</tr>
<tr>
<td>low</td>
<td>2,0</td>
</tr>
</tbody>
</table>

bank run

collusion of duopolists on trigger strategy

11.3 MULTIPLE EQUILIBRIUM

The consistency requirement of NE is weak.

Example: worker-firm again

<table>
<thead>
<tr>
<th>hard</th>
<th>easy</th>
</tr>
</thead>
<tbody>
<tr>
<td>high</td>
<td>2,1</td>
</tr>
<tr>
<td>low</td>
<td>0,0</td>
</tr>
</tbody>
</table>

Persuasion could also fail completely.

NE: both types say L, suppose 50-50 chance, the firm picks easy.
check

• Will the worker lie?

• Is the firm cheated?

it is like an er xin recycling, but it is consistent with rationality

11.4 EQUILIBRIUM SELECTION

the above un-informative equilibrium is unlikely in the long turn

all signals have the same strategic meaning, in particular H and L

but it’s hard for people to ignore the natural meaning of H and L.
especially when the interests align

but equilibrium selection could also depend upon the natural meaning of signals in more subtle ways

case 1: silence means “I won’t tell you”

<table>
<thead>
<tr>
<th></th>
<th>job1</th>
<th>job2</th>
<th>manager</th>
</tr>
</thead>
<tbody>
<tr>
<td>high</td>
<td>2, 5</td>
<td>1, −2</td>
<td>3, 3</td>
</tr>
<tr>
<td>low</td>
<td>1, −2</td>
<td>2, 5</td>
<td>3, 3</td>
</tr>
</tbody>
</table>

case 2: silence - high, anything else - low

The power to define signals matters.

Struggle over language might be long-term/short-term.

Farrell and Rabin (1996)

Lecture 12: Ultimatum and Trust game
Ultimatum game: proposer chooses to divide 10$, which the responder can veto, in which case no one gets paid.

Subgame perfect Nash equilibrium is offering 0.

Explanation of data: other-regarding preference

Trust game: A has 10$ to invest in B, with return rate of 200%. B divides the profit.

Subgame perfect Nash equilibrium is investing 0.

Holdup problem.

Explanation of data: intention or reciprocity, or ...
Research in gift-exchange labor market.

with labor surplus, shirking hard to monitor, paying a high wage (above market rate) is efficient.

find and policing – rational solutions – backfires.

Ford paying high.