1. Qiuqiu’s consumption violates WARP if $p_2x_1 < 1.5$ and $p_1x_2 < 1$. The probability of the former is 1/3 and the probability of the later is 1/2. So the probability of violating WARP is 1/6. (Message: Random behavior is often consistent with rationality.

2. $X$ is a finite set of objects, and $u$ is a real-valued function defined over $X$. For any $A \subseteq X$

\[ C(A) = \{ x \in A | u(x) + 1 \geq u(y) \forall y \in A/\{x\} \} . \]

Does $C$ satisfies the following two conditions? If so, prove it. If not, give a counter example.

(a) Condition $\alpha$. If $x \in B \subseteq A$ and $x \in C(A)$, then $x \in C(B)$.
Ans: Yes. Since $x \in C(A)$, $u(x) + 1 \geq u(y) \forall y \in B/\{x\}$.

(b) Condition $\beta$. If $x, y \in C(A)$, $A \subseteq B$ and $y \in C(B)$, then $x \in C(B)$.
Ans: No Example: $u(x) = 0, u(y) = 0.7, u(z) = 1.4. x, y \in C(\{x, y\}), y \in C(\{x, y, z\})$, but $x \notin C(\{x, y\})$.

3. Hanhan consumes only carrots and roasted pork. His utility function is

\[ u(c, r; \alpha) = \alpha c + (1 - \alpha) r \]

where $c$ and $r$ are the no. of units of carrots and roasted pork that he consumes, and $\alpha$ is a taste parameter that may vary from 0.4 to 0.6. This means that Hanhan likes roasted port relatively more when $\alpha$ is small. Let $X$ denote a finite set of consumption bundles (of carrots and roasted pork). For any subset $A$ of $X$, define

\[ C(A) = \{ (c, r) \in A | \exists \alpha \in [0.4, 0.6], u(c, r; \alpha) \geq u(c', r'; \alpha) \forall (c', r') \in A \} \]

as the elements of $A$ that Hanhan may like the best for some $\alpha$.

(a) Is it true that for any non-empty subsets $A, B$ of $X$ if $(c, r) \in C(A \cap C(B)$ then $(c, r) \in C(A \cup B)$? If true, give a proof. If false, a counter example.
Ans. False. For example: $(c, r) = (1, 1), A = ((1, 1), (1/0.4), 0)), B = ((1, 1), (0, 1/0.4)), (1, 1) \in C(A) as u(1, 1; 0.4) = u(1/0.4), 0; 0.4), (1, 1) \in C(B) as u(1, 1; 0.6) = u(0, 1/0.4; 0.6), but (1, 1) \notin C(A \cup B).

(b) Define a binary relation $W$ as follows: for any $(c, r), (c', r') \in X$

\[ (c, r) W (c', r') \text{ iff } (c, r) \in C((c, r), (c', r')) \]

Is $W$ transitive? If true, give a proof. If false, a counter example.
Ans: No. $(1, 1) W (0, 1/0.4)) and (0, 1/0.4) W (1.2, 1.2) as u(0, 1/0.4; 0.5) = u(1.25, 1.25; 0.5). But (1, 1) \notin C((1, 1), (1.25, 1.25))$
(c) Define a binary relation \( W^* \) as follows: for any \((c, r), (c', r') \in X\)

\[
(c, r) W^* (c', r') \text{ iff } (c', r') \notin C((c, r), (c', r')).
\]

Is \( W \) transitive? If true, give a proof. If false, a counter example.

Ans: Yes. Consider 3 consumption bundles \( a, b, \) and \( c \). If \( a W^* b \), then 
\[ (0.4, 0.6).a \geq (0.4, 0.6).b \text{ and } (0.6, 0.4).a \geq (0.6, 0.4).b. \]
If \( b W^* c \), then
\[ (0.4, 0.6).b \geq (0.4, 0.6).c \text{ and } (0.6, 0.4).b \geq (0.6, 0.4).c. \]
It follows that
\[ (0.4, 0.6).a \geq (0.4, 0.6).c \text{ and } (0.6, 0.4).a \geq (0.6, 0.4).c. \]